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Numerical analysis of stresses of a cracked cantilever beam subjected to distributed load

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Abstract

Cantilever beams are very commonly used equipment in structures. Increase in stress is a major reason for the failure of structures. When subjected to periodic load, structures undergo various changes in their principal and shear stress values. When these stress values reach a threshold point, the structure collapses. In order to prevent failure, it is necessary to know about the stress values of cantilever beams. Previously various researchers have studied the stress intensities and stress distribution of various types of structural elements such as beams, columns. But their study didn't include the presence of a crack. In this study, the changes in both principal and shear stress values for cracks positioned at various locations, for different crack depth and crack opening size has been studied. The commercial simulation software 'Abaqus CAE' was used and for crack modelling 'Solidworks' software was used. A combination of hexahedral and wedge elements was used for better accuracy around the cracked portion. The results indicate that as the crack moves closure to the fixed end the increase in stress value near the cracked region increases. The stress values near the cracked region also increase for a larger crack opening and a larger crack depth.

Keywords: Principal Stress, Shear Stress, Stress Concentration, Finite Difference Method.

1. Introduction

When a beam is subjected to a force, an internal force is also applied to the plane of action of external force. This internal force per unit area of that plane is called stress. The force which is subjected to a plane of a beam can be applied either perpendicularly or parallel. The force acting perpendicularly is called normal force (It can be off tensile or compressive type).[1] The force acting parallel is called shear force or tangential force. If a plane of a beam is only subjected to a normal force with no shear force, the plane is called principal plane. The stress acting on a principal plane is principal stress. It gives information about a location under crack in a stressed beam. Principal stress involves a vector resolution of a stress state so that the shear stress is reduced to zero. It is a very effective and useful procedure to obtain knowledge of strength about any stressed beam. In the presence of a crack there is a change in nodal stress values in the structures and it plays a vital role in the life cycle of the structure. Therefore, it is necessary to identify the effects of crack on structures in order to prevent failure. Previously various researchers have worked with this study. Y Narkis [2] studied the dynamics of a simply supported beam under crack. He first simulated the beam and using these simulated data he built a mathematical model from which crack location can be studied. Yin Zhang[1] studied the three models of surface stress for three different conditions and then compared these models with three different scenario. He also studied the stiffening effects of the tensile surface stress. Wahalathantri[3] presented a material model to simulate load induced cracking in RC elements. He studied the stress strain behavior of concrete under compression and tension damage properties.He used ABAQUS for his analysis. The model developed by Nayal and Rasheed (2006) is selected for the present study as it is applicable for both reinforced and fibre reinforced concrete with only minor changes. Also, this method indicates similarity to the tension stiffening model that is needed for ABAQUS concrete damaged plasticity model. This tension stiffening model was originally based on the homogenized stress-strain relationship developed by Gilbert and Warner (1978) which accounts for tension stiffening, tension softening and local bond slip effects.

The stress of a beam subjected to a crack is the indication of the probability of the structure failure. This paper presents a numerical comparison among the results of the stress calculation in different location. A beam of cantilever type was considered as a specimen. The stress in the different location of the beam was calculated using numerical methods. Different types of cracks in different depth with different opening have been introduced in these locations. The change in stress was measured. The stress are calculated to be high if the cracks are present in a location of the beam closer to the fixed enf. When the stress are high in a location, the probability of structure failure is more. For each case, the stress induced was calculated. For the different location in different depth with different opening, the results are compared. The software, abaqus finite element package, was used in the calculation of the stress in cantilever beam.The results clearly indicated that as the crack moved closer to the fixed end,as the crack depth and width increases ,the nodal stress value at the tip of the crack increased.

2. Numerical Methodology

Based on the fashion of physical conditions problems can be divided into three types.
1.Initial Value Problem
2.Boundary Value Problem
3.Initial-Boundary Value problem
Basic Numerical approach for boundary value problems are:
1.Finite Difference Method
2.Finite Element Method.
3.Finite Volume Method.
4.Boundary element Method.
Basic steps required to solve BVP's by FDM are
1.Discritization of computational domain
2.Conversion of DEQ into equivalent algebraic equations.
3.Application of the governing DEQ for nodal points other than boundary nodes.
4.Application of boundary conditions for boundary nodes.
5.Construction of global co-efficient matrix equation which ensures no of equations are equal to the number of unknowns.
6.Numerical Solution to the GCME by either elimination or iteration schemes.
7.Post processing of the primary FD solution to calculate any other parameter associated with the physical systems.

General Governing equations

For deflection and moment of beams following equations can be used

$$\frac{d^2}{dx^2}\left(EI\frac{d^2y}{dx^2}\right)+p\frac{d^2y}{dx^2}=q(x) \quad (1)$$

Where,

$$\frac{d^2y}{dx^2}=\frac{y_{i-1}-y_i+y_{i+1}}{\Delta x^2}+0((\Delta x)^2) \quad (2)$$

For a beam with one end fixed the following equations are used for calculating bending moment and shear

$$M=EI\frac{d^2y}{dx^2} \quad (3)$$

$$V=EI\frac{d^3y}{dx^3} \quad (4)$$

3. Methodology

Geometry

There can be various types of cantilever beam depending on the geometry of the cross section. Here in this study a rectangular shaped cross section has been used for designing the cantilever beam

Table 1. Detailed dimension of cantilever beam

Property	Value
Depth	0.20 m
Width	0.25 m
Length	3 m

Crack Design

A wedge shaped crack was used to analyze the behavior of vibration under cracks. The cracks were designed in solidworks 2017 and then the files were imported into Abaqus CAE.Three different openings of 0.002m,0.004m,0.010m were used. Cracks were designed at 0.7 m distance interval from the free end. Here crack opening of 0.002,0.004,0.006 and 0.010m were used and crack depth of 0.025,0.0375,0.050,0.0625 and 0.075m were used.

Material Selection

For the vibration analysis mild steel was used as a material because they are very frequently used in structural elements.

Table 2. Material properties

Property	Value
Modulus of elasticity	210 Gpa
Mass density	7860 kg/m3
Poisson ratio	0.3
Applied load	1000KN

Meshing

For accurate analysis of the beam the area around the crack was partitioned and the tip of the crack was divided from the rest of the region with a circle and Wedge shaped elements were used near the crack and element density was increased for a better analysis.

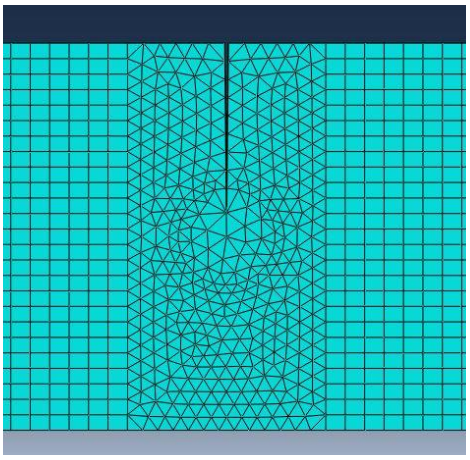

Figure 1. Meshing of the cantilever Beam

4. Result & Discussion

This Section is dedicated to compare and discuss of results based on various parameter.

Based on Crack Location

The path along which the stresses are going to be analyzed is shown in figure 2

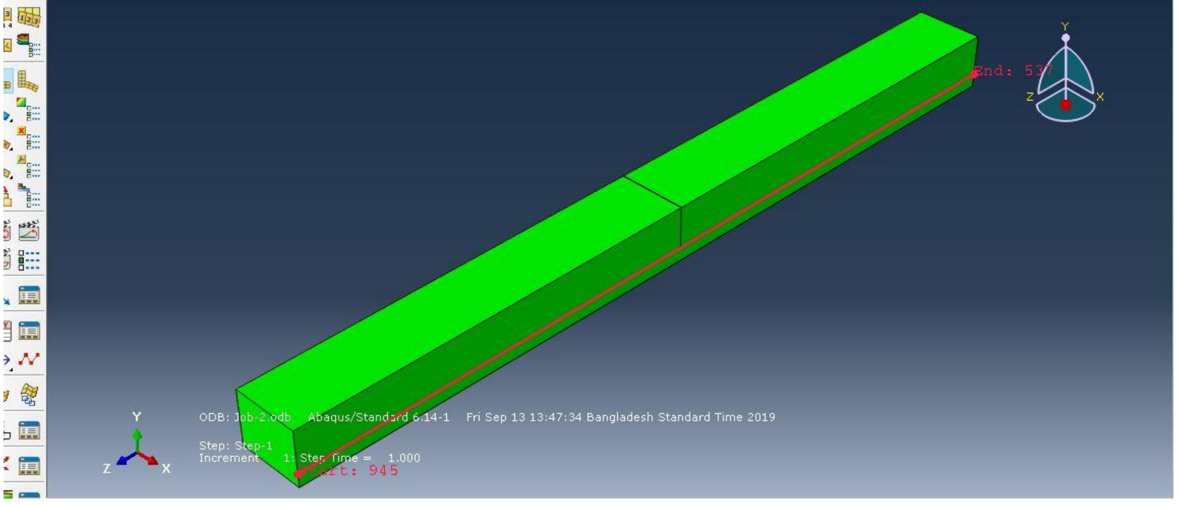

Figure 2. Selected path along z axis

This path starts from a node at the fixed end and goes in such a way that it is tangential to the tip of the crack so that the maximum effect of the crack can be analyzed.The fluctuation in σ_{zz} for various crack location along the z axis is given in figure 2

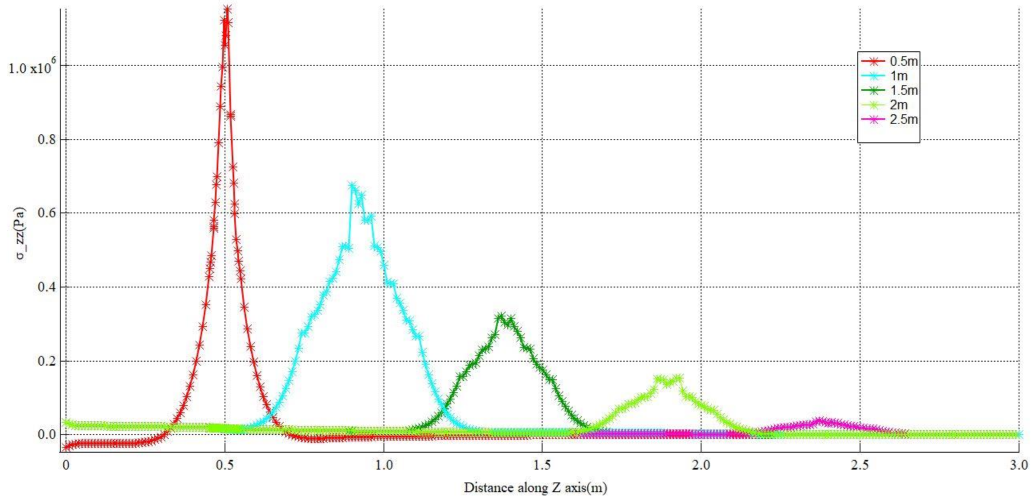

Figure 3. σ_{zz} distribution for various crack location

The figure clearly indicates that as the crack moves closer to the fixed end of the beam the stress intensity also increases.This means that if the crack is located closer to the fixed axis then the probability of failure is higher. This occurs due to the fact that elements near the fixed end hold majority of the stress component values.

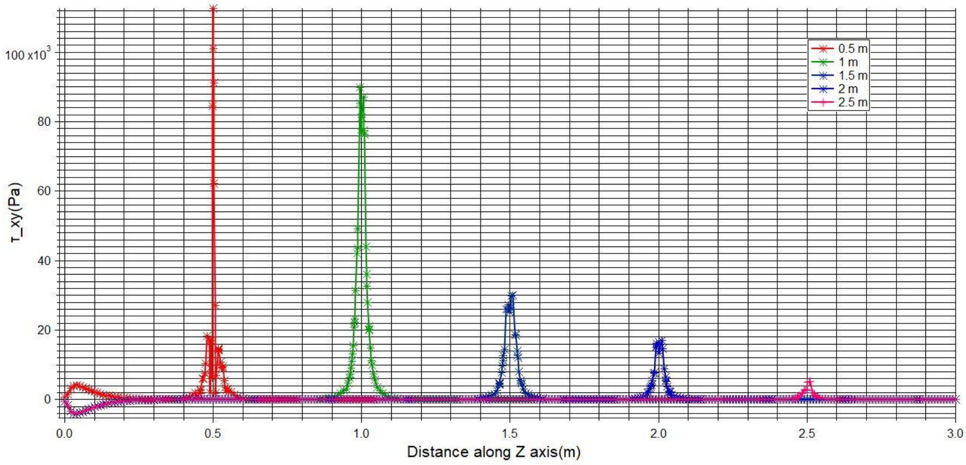

Figure 4. τ_{xy} distribution for various crack location

Similar to the previous figure in this figure it can be seen that the shear stress also increases as the crack moves closure to the. Here nodal values have been taken to analyze the results. Here it is clearly evident that the nodal shear stress value is maximum at the element which is located at the tip of the crack and if failure occurs then the first impact will start from the tip of the crack and then it will spread slowly. It is evident if force is applied on the structure then the crack growth will start from the tip of the crack which will ultimately result in the failure of the structure.

Based on Crack Depth

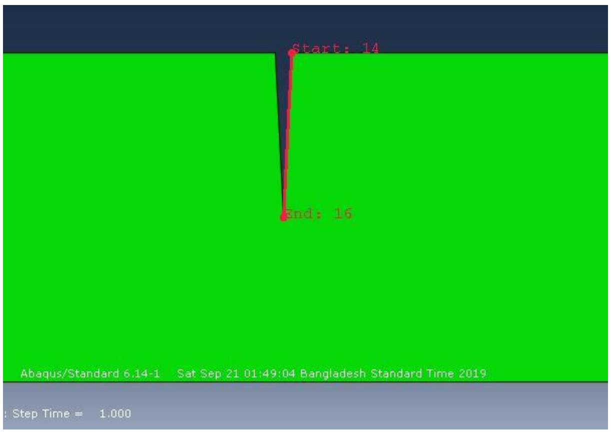

Figure 5. Path along the crack

For a better understanding of the effect of crack depth and width, a path had been created along the crack and nodal values were measured from that path.

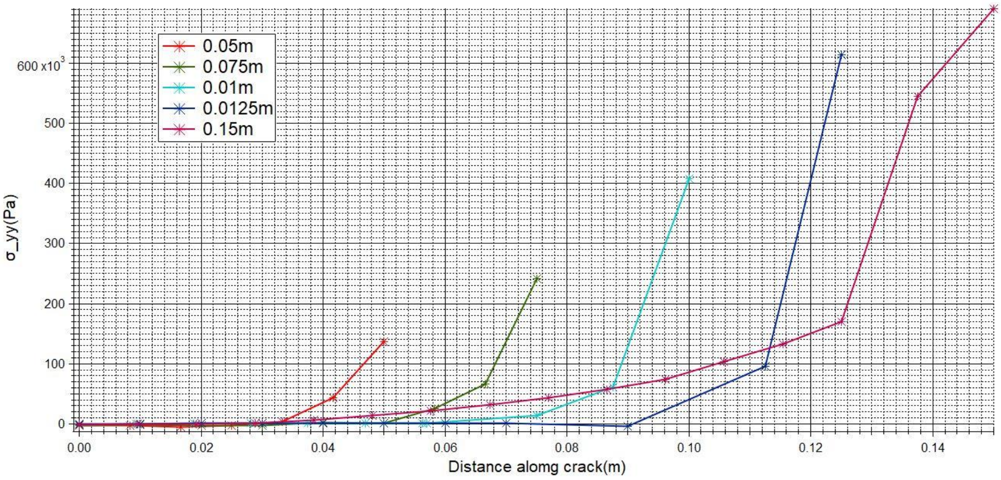

Figure 5 .σ_{yy} distribution for various crack depth

In this case σ_{yy} distribution has been measured along the crack depth. Here σ_{yy} is selected because the values were taken from node points which lied on the xy plane. It is clearly evident from the figure that as the crack depth increases the value of principal stress also increase. It denotes that if the crack depth is larger than the crack propagation chances are higher and also the chances of structural failure is also on the upper percentile range

Based on Crack opening

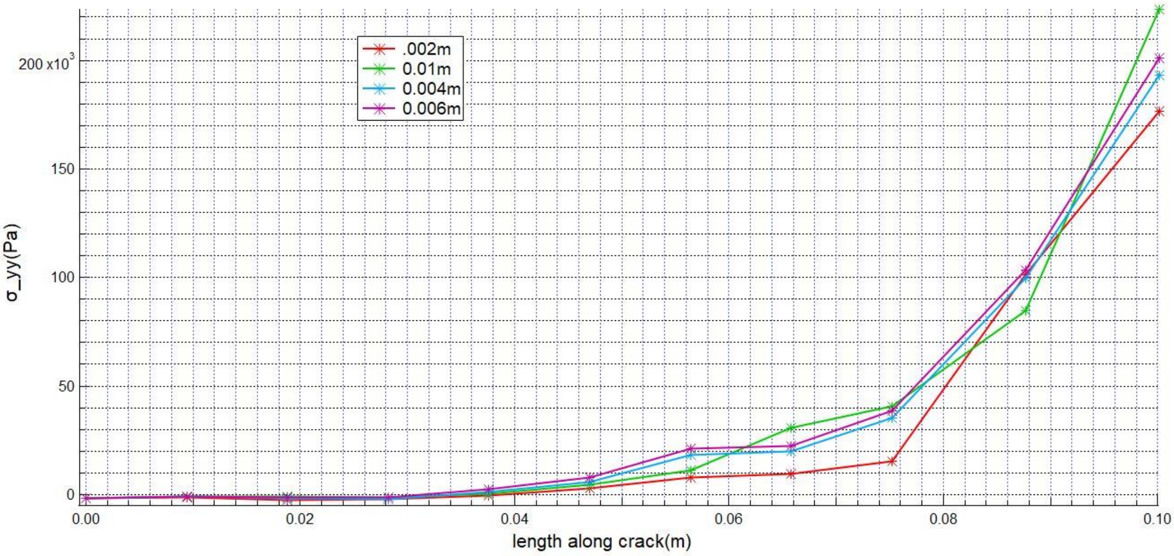

Figure 6. σ_{yy} distribution for various crack opening

In this analysis just like the case of crack depth, principal stress along the crack is measured for various crack opening. It can be seen from the graph that as the crack opening increases the stress values at the tip of the crack increases. And it is known that a structure is most vulnerable at the tip of the crack. So it is a clear indication that if crack opening increases the chances of failure of structure also increases.

5. Conclusion

The effect of crack is very much significant in stress analysis. Following results has been obtained from this study

- In the presence of crack nodal stress value is maximum at the tip of the crack.
- As the crack moves closer to the fixed end of the cantilever beam, the nodal stress values also increase.
- With larger crack opening the principal stresses are larger.
- As the crack moves deeper the nodal stress values also increases.

6. References

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